The most commonly used correlation between shear stress (τ) and shear rate (γ) is the power law expression

$$\tau = \eta\gamma^{n} \qquad (A)$$

where η is the 'consistency coefficient' and *'n'* is the flow index. For laminar flow the pressure drop is related to the flow rate *via* Wilkinson's equation

$$\frac{\Delta P}{L} = \frac{2\eta}{r}\left[\frac{(3n+1)Q}{\pi r^3 n}\right]^n \qquad (1)$$

where Q is the volumetric flow rate and r is the pipe radius.
The generalised Reynolds number (Re*) is

$$\mathrm{Re}^* = \frac{8\rho v^{2-n} d^n}{\eta(6+2/n)^n} \qquad (2)$$

For turbulent flow the friction factor is

$$f^{-1/2} = \frac{4}{n^{0.75}}\log\left[\mathrm{Re}^* f^{(1-n/2)}\right] - \frac{0.4}{n^{1.2}} \qquad (3)$$

and, as always,

$$\frac{f}{2} = \frac{\Delta P d}{4L\rho v^2} \qquad (4)$$

i). Rheological tests on 'milk of magnesia' have provided the following data:

shear rate (s^{-1}):	7.2	16	64	320	720
shear stress (Pa):	7.0	9.1	14.3	24.2	31.6

Calculate the consistency coefficient and the flow index.
Ans: 'flow index' of 0.327 and 'consistency coefficient' of 3.67
N.B. Linearise equation (A) by taking logs and plot above data on a log-log plot; use the gradient and intercept for the flow index and consistency coefficient respectively.

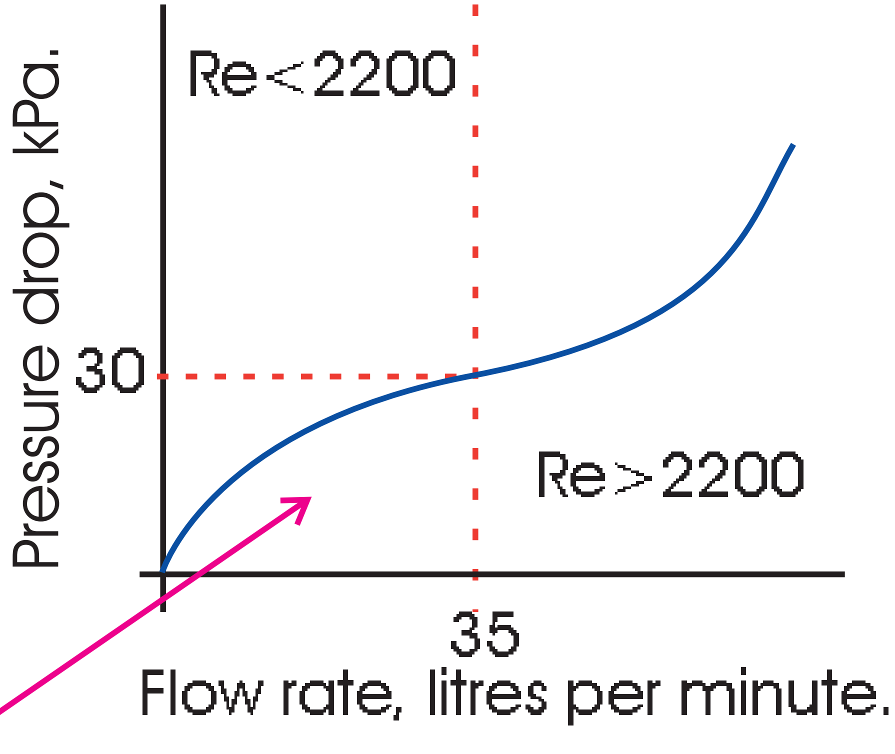
ii). During the production of milk of magnesia the suspension must be pumped 2.11 m down a pipe of radius 6.95 mm. The mean suspension density is 1300 kg m^{-3}. Complete the following table that describes the flow-pressure dependency.

For Laminar flow:				**For Turbulent flow:**			
Pressure drop	Flow rate	Velocity	Re*	Velocity	Re*	$f^{-1/2}$ equn(4)	RHS of equn(3)
(kPa)	($m^3\ s^{-1}$)	($m\ s^{-1}$)		($m\ s^{-1}$)			
5	$2.1\text{x}10^{-6}$	0.014	--------	--------	--------	--------	--------
15	$5.9\ \text{x}10^{-5}$	0.39	--------	--------	--------	--------	--------
30	$4.9\text{x}10^{-4}$	3.24	2200	--------	--------	--------	--------
40	$1.2\text{x}10^{-3}$	7.82	9700	4.66	4060	14.64	14.64
80	$9.9\text{x}10^{-3}$	65.1	--------	7.64	9300	17.0	17.0

N.B. the laminar data comes from equation (1), but data at 40 and 80 kPa are at too high velocity. So, starting with the laminar velocity equations (2), (3) and (4) are used – reducing the velocity by some amount, until equations (4) and (3) balance: see last two columns above.

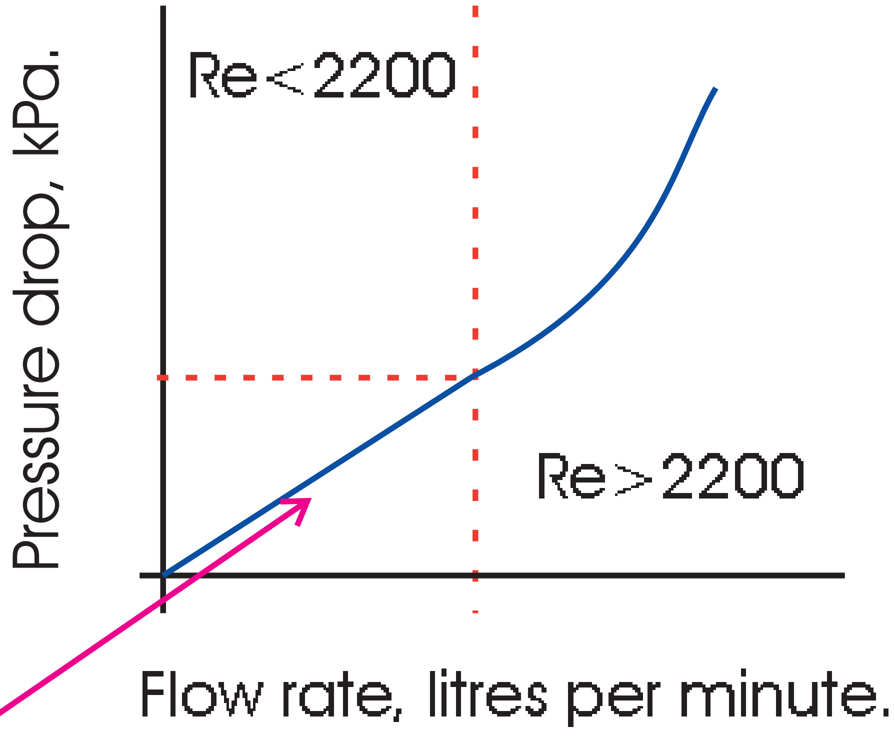
See the two graphs on the next page

FLOW CURVE FOR MAGNESIA:



Laminar flow region is not linear - shear thins

FOR A NEWTONIAN MIXTURE:



Laminar flow region is linear