

4 Filtration of liquids

Filtration is the removal of suspended particles from a fluid, performed by a filter medium, septum, cloth or bed of solids. In this chapter we will discuss only liquid filtration, the removal of particles from gases is covered in Chapter 14. Filtration is commonly encountered in chemistry laboratories on a Buchner funnel and within the kitchen during the making of filter coffee. It is a very important industrial process as it is often a key stage in product recovery: following reaction, precipitation and crystallisation stages, but preceding thermal drying and packaging (e.g. in pharmaceutical production). It is more economic to remove moisture from particles by mechanical means, including filtration, than by thermal means. Thus, domestic washing machines provide higher and higher spin speeds prior to thermal, or evaporative, drying.

There is a vast range of filtration types; depending upon whether the objective is to produce a clean liquid, as in drinking water production, or solids retained in a filter cake, as in product recovery. The former process is called clarification, or clarifying filtration and is often performed in equipment containing packed beds, which were introduced in the previous chapter.

4.1 Deep bed and clarifying filtration

Deep bed filtration uses packed beds of particles between 300 and 5000 μm in diameter and the bed height is usually between 0.5 to 3 metres. Deposition of suspended solids takes place within the bed, by a variety of particle adhesion and collection mechanisms. Some are discussed in Chapters 13 and 14. The most pertinent are: sedimentation (on to the bed grains), inertia, van der Waals, diffusion, electrostatic attraction and repulsion. Within the bed the fluid flow condition is predominantly laminar – to minimise the scouring effect of turbulences. However, the packed bed will eventually contain a large amount of deposited solids and need regenerating; at this stage a *backflush* is often employed, which may involve inducing fluid turbulences. The frequency of regeneration will depend upon the concentration of solids in suspension entering the bed. This is usually low: less than 0.5 grams per litre. The feed flow rate is usually less than 8 m^3 per m^2 of bed area per hour and it is possible for the bed filter to reduce the outlet (effluent) solids concentration to below 0.1 milligrams per litre. However, when processing very small particles, such as bacteria and viruses, the feed rate may need to be low: $0.1 \text{ m}^3 \text{ m}^{-2} \text{ h}^{-1}$, in order to provide the effluent quality required; i.e. particle removal.

As particles are deposited within the packed bed, the resistance to fluid flow increases. If the bed is running under a constant pressure difference the resulting flow rate will diminish, as illustrated in

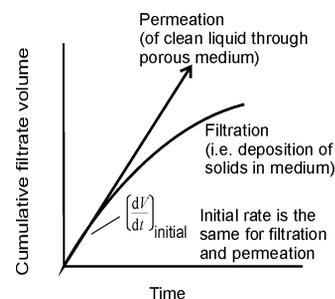


Fig. 4.1 In a filter increasing resistance causes liquid flow to decrease with time – at constant pressure drop

Uses for bed filters

Drinking (potable) water, pretreatment for high purity water, e.g. electronics industry, effluent treatment, beer and wine clarification, sea water filtration before injecting into oil reservoirs.

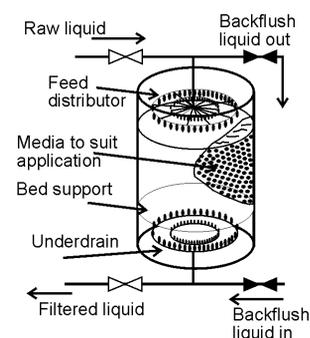


Fig. 4.2 A deep bed filter

History

In the UK, Dr John Snow first traced a water borne cholera epidemic to a public water pump in Broad Street, London, in 1854. Surrounding areas not using the pump were unaffected. The connection between water and disease led to mass deployment of sand filters (300 μm grains and 0.75 m high), which were previously just considered to improve the water look and taste. A public house, called the John Snow, now stands on the site of the old Broad Street pump.

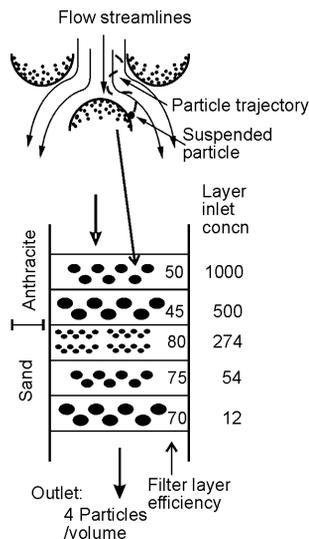


Fig. 4.3 Illustration of mixed media deep bed with cut-away showing particle deposition

Figure 4.1. When the filtrate rate drops to an unacceptably low value, the filter is backflushed at a rate of up to $36 \text{ m}^3 \text{ m}^{-2} \text{ h}^{-1}$ and may include an air scour and some degree of fluidisation (Chapter 7). So, for continuous production a second filter is required; so that whilst one is filtering the other can be cleaning and put on stand-by. In general, backflushing uses up to 5% of the previously filtered liquid. In some applications the backflush cycle is timed and, for constant rate filtration – where the pressure drop increases to maintain the rate, the backflush is initiated by a pressure threshold. Figure 4.2 shows a schematic illustration of a bed filter with the associated flow lines for operation.

Various media may be used as the bed, including: sand, gravel, anthracite, activated carbon and garnet. The carbon based materials provides removal of some dissolved species as well as filtration of suspended matter from the liquid. In many instances a mixed bed is used: coarse particles of one type on top of fine particles of another. One advantage of this arrangement is that, if the coarse particles treat the down-flowing suspension first, they will remove the coarser suspended solids – the finer material will pass between the coarse grains, but will become captured on the finer grains in the lower bed. This segregation of filter media prevents the finer suspended material from clogging up the gaps at the top of the filter bed. However, cleaning by backflushing would normally result in the finer filter grains transferring to the top of the bed, but this can be overcome by designing the bed to contain dense fine grains (of say sand) and coarse light grains (of say anthracite). An illustration of a mixed media bed is given in Figure 4.3. Filtration performance is also improved by the use of flocculants and coagulants (Chapter 13), prior to passage through the bed, to increase the size of the suspended particles to be captured within the media.

The basic design, or modelling, of deep bed filters can be achieved using a simple equation

$$\frac{dN}{dz} = -\lambda N \tag{4.1}$$

where λ is a filtration constant (m^{-1}). Unfortunately, the filtration constant will change with time, as material becomes deposited within the bed; hence, equation (4.1) can only be treated as an ordinary differential equation at the start of the filtration and integrated to give

$$N = N_0 \exp(-\lambda_0 L) \tag{4.2}$$

where λ_0 is the initial filtration constant, L is the depth within the filter and N is the concentration of solids in the influent (on any consistent basis: mass per volume, volumetric, etc.). Thus, the concentration of solids deposited from the influent suspension exponentially decays within the bed. It is possible to model the performance of a bed filter by allowing the filtration constant to become a variable dependent upon the following

$$\lambda = f(\varepsilon_s \varepsilon_0 \sigma \lambda_0) \tag{4.3}$$

where ε_s is the surface deposit porosity (of the material deposited on the grains), ε_0 is the packed bed porosity of the filter grains and σ is the specific deposit: the ratio of the volume of solids deposited within a layer to the volume of the filter layer.

Apart from the filtration performance, the pressure drop during filtration is important. This can be deduced from the Kozeny-Carman equation, but it is usual to combine it with the hydrostatic head equation derived from Bernoulli's equation, thus

$$\frac{dh}{dz} = \mu \left[\frac{K(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] U_0 \frac{1}{\rho g} \quad (4.4)$$

where h is pressure in terms of *head of liquid*. Strictly speaking, the porosity is the porosity remaining open to flow; which will be that formed from the initial bed voids and what is left after deposition of solids. However, the latter effect is normally limited so the initial bed porosity is used. However, the pressure drop, or head, will increase during filtration as the flow channels become smaller, or clogged. Hence, equation (4.4) is only safely applied towards the beginning of a bed filtration. The equation can be integrated over the full bed height to give the overall head requirement.

A more recent development from the batch bed filter design is the continuous sand filter. The sand bed is continually moving downwards within the vessel because the bed at the bottom enters an air lift pump during which deposited solids are scoured off the sand grains. At the top of the air lift the sand quickly sediments to rejoin the top of the bed, but the deposits on the sand grains are *elutriated* (i.e. washed) out through a separate discharge pipe. Thus, there is no need for backflushing and the consequent taking the filter off-line.

Deep bed filtration is just one example of clarifying filtration, where the objective is to remove suspended solids at a low concentration from a liquid. Other filter types that can be used for this include: membranes, pressure leaf filters, candle filters and pre-coat filtration, see Section 4.8. Excepting pre-coat filtration, the intention is to provide a filter medium capable of retaining the particles in suspension without clogging, or providing high flow resistance. A set of candle filters are illustrated in Figure 4.4 and, typically, filtration would take place at up to 6 bar gauge pressure.

4.2 Cake filtration

When the solid concentration in the suspension is sufficiently high, solids will *bridge* over the entrance of the filter medium pores. This is cake filtration and it is illustrated in Figure 4.5. The minimum slurry concentration required for cake filtration depends on the nature of the solids and filter medium, but is usually about 0.5% by volume. The modelling of cake filtration assumes that Darcy's law can be applied to both the filter medium and the forming filter cake. The assumption is that the two resistances to fluid flow, or their resulting pressure drops, can be added together similarly to Figure 3.2, thus

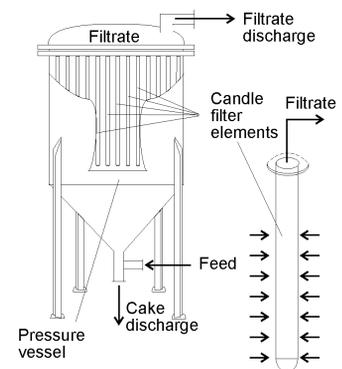


Fig. 4.4 Candle filtration: the high surface area provided by the filters in parallel helps provide good filtrate rates. Filter cleaning is required on a periodic basis. Filtration is usually performed under constant rate conditions.

pictures

For illustrations of equipment described here see:
www.solidliquid-separation.com

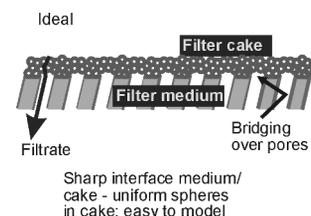


Fig. 4.5 Bridging of particles over the surface openings of a filter medium – thus new cake is formed on top of old layers of cake and it is possible for the medium openings to be much bigger than the particles

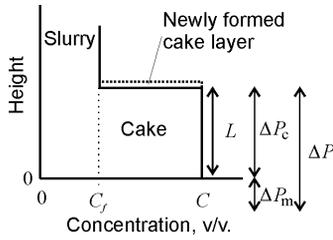


Fig. 4.6 The filter cake concentration step change from the feed concentration C_f to the cake concentration C

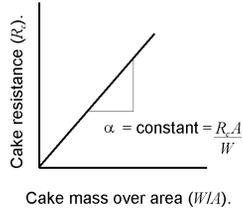


Fig. 4.7 The variation of cake resistance with mass of cake deposited – definition of specific cake resistance

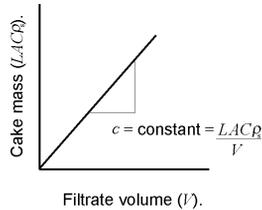


Fig. 4.8 The variation of cake mass with filtrate volume – definition of dry cake mass per unit filtrate volume

$$\Delta P = \Delta P_c + \Delta P_m \tag{4.5}$$

where ΔP is the total applied pressure (i.e. over both the cloth and cake), whereas ΔP_c is pressure drop over the cake and ΔP_m is pressure drop over the filter medium (normally a cloth). Applying Darcy’s law to the pressure drops, equation (3.4), gives

$$\Delta P = \frac{\mu L}{A k} \frac{dV}{dt} + \frac{\mu L_m}{A k_m} \frac{dV}{dt} \tag{4.6}$$

where the subscript ‘m’ denotes the medium and L represents the cake height at any instance in time. The medium height and permeability will be essentially constant during the filtration and can, therefore, be replaced by a constant medium resistance term (R_m). Likewise, the cake resistance (R_c) can be defined as follows but, unlike the medium resistance, it will increase as the cake height increases

$$R_m = \frac{L_m}{k_m} \quad \text{and} \quad R_c = \frac{L}{k}$$

Substituting the resistance terms into equation (4.6) and rearranging

$$\frac{dt}{dV} = \frac{\mu}{A \Delta P} (R_c + R_m) \tag{4.7}$$

Figure 4.6 illustrates the occurrence of a new layer of cake: the concentration jumps from C_f to that of the cake C ; the liquid displaced by the solids concentration increase must be sucked, or pushed, through the layers of previously formed cake. Hence, it is appropriate to apply Darcy’s law for pressure the drops due to: flow through the layers of previously formed cake and through the filter medium. In equation (4.7) the medium resistance term is a constant, but the cake resistance term increases with deposit thickness.

4.3 Specific cake resistance and dry cake mass per filtrate volume

Equation (4.7) has too many variables to be solved: time, volume, R_c and, under certain circumstances, pressure drop. The cake resistance must be related to one of the other variables. This is possible because it is the increasing cake depth that causes R_c to vary and, for incompressible cake filtration, there will be a constant volume of cake deposited per volume of filtrate. Hence, L can be related to V , enabling the substitution to be made. It is formally achieved as follows. Figure 4.7 illustrates that the cake resistance increases proportionally with increasing cake mass, or mass per unit filter area. The constant of proportionality is called the *specific resistance* to filtration (α) and it is similar to an equivalent length used for fittings in pipe flow calculations; e.g. an extra length of pipe in resistance terms to account for the extra pressure drop due to the fittings. Thus, the equation is

$$R_c = \alpha W / A \tag{4.8}$$

where W is the total mass of cake deposited, which can be related to the volume of cake (LA) by the volume fraction of solids in the cake (C) and the solids density. See Figure 4.6 for an illustration of cake solids volume fraction and cake height. Thus

$$W = LAC\rho_s \tag{4.9}$$

and, for an incompressible cake filtration, the cake mass will uniformly increase with the filtrate volume. This is illustrated in Figure 4.8. The constant of proportionality is called the *dry cake mass per filtrate volume* and has the symbol c . Thus

$$c = \frac{LAC\rho_s}{V}$$

which can be rearranged to give

$$LAC\rho_s = cV \quad \text{hence from equation (4.9)} \quad W = cV$$

which is substituted into equation (4.8) to give

$$R_c = \alpha cV / A \tag{4.10}$$

For the filtration of a given material, at a constant feed concentration, both specific resistance and dry cake mass per filtrate volume will be constants (they are the constants of proportionality in Figures 4.7 and 4.8); hence, cake resistance varies proportionally with filtrate volume. Substituting equation (4.10) into (4.7) provides

$$\frac{dt}{dV} = \frac{\mu\alpha c}{A^2\Delta P}V + \frac{\mu R_m}{A\Delta P} \tag{4.11}$$

which is the *general filtration equation* and is valid for all types of incompressible cake filtrations. It has three variables: time, volume and pressure drop. All the remaining terms are constant for the filtration of a specific material at constant concentration of the feed slurry. The specific resistance is a property of the particles in suspension, but not the feed concentration; whereas the dry cake mass per filtrate volume is both a property of the particles in suspension and the feed concentration employed. Equation (4.11) is solved under specific conditions: constant pressure, constant rate or both varying – as described in Section 4.5

4.4 Compressible cake filtration

Figure 4.9 illustrates what is meant by compressible cake filtration: the cake concentration, and hence the permeability and specific resistance, are a function of the pressure drop applied during the filtration. It is logical that an increase in concentration increases the resistance to fluid flow within the cake and it is possible to show that the specific resistance and cake permeability are simply related. Starting with the original definition of cake resistance and substituting equation (4.8)

$$R_c = \frac{L}{k} = \frac{\alpha W}{A}$$

Using equation (4.9) and rearranging for specific resistance

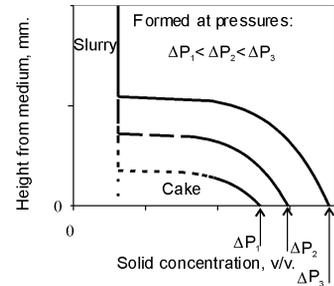


Fig. 4.9 Illustration of compressible cake filtration, increasing concentration of the cake with applied pressure difference – compare with Figure 4.6, where C is independent of cake forming pressure.

Recommendation
 Leave the compressible cake filtration section until the problems at the end of this chapter have been completed.

Filtration modelling

Permeability can be deduced from a particle size distribution – hence specific resistance from (4.12), so long as the cake concentration can be estimated. Likewise, equation (4.14) provides a value for c . Both can then be used in (4.11) for filtration modelling. However, laboratory filtrations used to scale-up filtration data are much more reliable.

Filtration constants

The empirical constants tell us about the nature of the material to be filtered:

vacuum filtration region

$$n < 0.3 \text{ and } \alpha < 1 \times 10^{11} \\ m < 0.1$$

pressure filtration region

$$0.3 < n < 0.6 \\ 1 \times 10^{11} < \alpha < 2 \times 10^{12} \\ 0.05 < m < 0.3$$

See:

www.filtration-and-separation.com for a list of measured values for common materials.

$$\alpha = \frac{AL}{kLAC\rho_s} = \frac{1}{kC\rho_s} \quad (4.12)$$

and the permeability can be provided by the expression given on page 24, which demonstrates that it is a function of porosity (or cake concentration by volume fraction), as well as specific surface of the particles. Likewise, the dry cake mass per filtrate volume will also depend on the applied pressure, again this is because the cake concentration increases with applied pressure. The relation between dry cake mass per filtrate volume is derived from a mass balance on solids and liquid entering the filter as follows.

Total mass slurry filtered	= M	(kg)
Mass fraction of solids in slurry	= s	(-)
Mass of solids in slurry	= sM	(kg)
Total mass of liquid in slurry	= $(1-s)M$	(kg)
Mass liquid retained in filter cake	= $(1-C)AL\rho$	(kg)

So, provided that all the solids are retained in the filter cake, the dry cake mass per filtrate volume (N.B. volume liquid in the filtrate and not total liquid volume) is

$$c = \frac{sM}{[(1-s)M - (1-C)AL\rho] / \rho} \quad (4.13)$$

The solid mass can be represented in terms of both the solids retained in the cake and the original solids in the slurry

$$CAL\rho_s = sM \quad (kg)$$

which can be rearranged to give

$$AL = \frac{sM}{C\rho_s}$$

and substituted in to equation (4.13)

$$c = \frac{sM\rho}{[(1-s)M - (1-C)sM\rho] / C\rho_s} = \frac{1}{(1-s)/(s\rho) - (1-C)/(C\rho_s)} \quad (4.14)$$

So, with compressible cake filtration, both the specific resistance and dry cake mass per filtrate volume are functions of the filtering pressure because it influences the cake concentration. There are several expressions relating the average specific resistance and average cake concentration (by volume fraction) to pressure difference; the easiest to apply are

$$\bar{\alpha} = \alpha_0(1-n)\Delta P_c^n \quad (4.15)$$

and

$$\bar{C} = C_0(1-m)\Delta P_c^m \quad (4.16)$$

where n , m , α_0 and C_0 are empirical constants. The coefficient 'n' is sometimes referred to as the compressibility coefficient and the greater the value the greater the compressibility. A value of zero indicates incompressible solids, calcite is about 0.2, kaolin clay about 0.6 and fine sludges approach unity. Unfortunately, equation (4.15) is unusable when the compressibility coefficient approaches unity.

The average dry cake mass per filtrate volume can be deduced from the average cake concentration by equation (4.14).

$$\bar{c} = \frac{1}{(1-s)/(s\rho) - (1-\bar{C})/(\bar{C}\rho_s)} \quad (4.17)$$

In equations (4.15) and (4.16) the pressure forming the filter cake should be used; this will be less than the total applied pressure drop because some of the applied pressure will be used to force the fluid through the filter medium. However, the use of a higher pressure difference in equation (4.15), than really exists, will result in a higher specific resistance to filtration which is a safer design option than under-estimating filtration resistance. The actual difference between total applied pressure and that over the cake is normally very small.

4.5 Filtration modes of operation

Equation (4.11) contains three variables: time, volume and pressure difference. It can be solved by holding one of these variables constant. Considering *constant pressure filtration*, the equation can now be integrated from time zero to some time t , during which the volume passing through the cake (and medium) changes from zero to V - which is the total filtrate volume at time t . Integrating and rearranging provides

$$\left(\frac{\mu\alpha c}{2A^2\Delta P}\right)V^2 + \frac{\mu R_m}{A\Delta P}V - t = 0 \quad (4.18)$$

which is a quadratic equation in terms of filtrate volume. Thus, if all the filtration constants are known and the filter cake formation time (t), it is possible to solve equation (4.18) for the filtrate volume. In laboratory tests it is usual to operate at constant pressure, employing a constant vacuum filter such as a Buchner type, and equation (4.18) can be rearranged to assist data analysis to give

$$\frac{t}{V} = \frac{\mu\alpha c}{2A^2\Delta P}V + \frac{\mu R_m}{A\Delta P} \quad (4.19)$$

which is the equation of a straight line, as illustrated in Figure 4.10. The gradient can be used to deduce the specific resistance and the filter medium resistance contributes towards the intercept; the other constants include: fluid viscosity, filter area and total pressure difference. However, the dry cake mass per filtrate volume must be independently determined, as this is also a contributor towards the gradient. This is normally achieved by taking a cake sample and weighing it, followed by drying then reweighing when dry. This provides the *moisture ratio*

$$m_R = \frac{\text{mass of wet cake}}{\text{mass of dry cake}} \quad (4.20)$$

Extending the material balance just before equation (4.13):

Total mass of liquid in slurry	= $(1-s)M$	(kg)
Mass of <i>dry</i> cake	= sM	(kg)
Mass of <i>wet</i> cake	= sMm_R	(kg)
Mass of liquid retained in the cake	= $sMm_R - sM$	(kg)
	= $(m_R - 1)sM$	(kg)

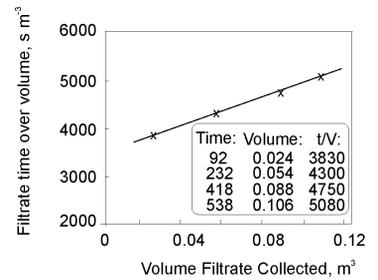


Fig. 4.10 Data analysis by equation (4.19)

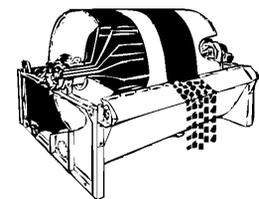
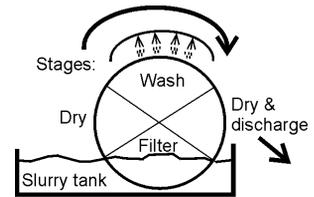


Fig. 4.11 Schematic diagram of a Rotary Vacuum Filter (RVF)

So, provided that all the solids are retained in the filter cake, the dry cake mass per filtrate volume (i.e. volume liquid in the filtrate and not total liquid volume) is

$$c = \frac{sM}{[(1-s)M - (m_R - 1)sM] / \rho} = \frac{sM\rho}{(1-s)M - (m_R - 1)sM}$$

which simplifies to

$$c = \frac{s\rho}{1 - m_R s} \tag{4.21}$$

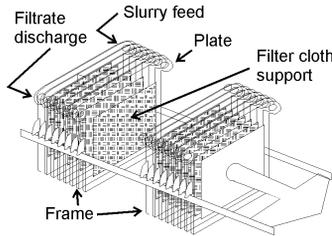


Fig. 4.12 Schematic diagram of a plate and frame filter

Hence, a knowledge of the moisture ratio, slurry solids concentration by mass fraction and liquid density provides the dry cake mass per filtrate volume and the gradient from a plot similar to Figure 4.10 will provide the experimentally determined value for specific resistance to filtration. If a series of tests are conducted, at different total filtration applied pressures, then it is possible to assess if the resistance varies with pressure: a log plot of these is suggested by equation (4.15), with the gradient providing the exponent in the equation and the intercept can be used to deduce α_0 . A similar plot is used for concentration variation with pressure, in accordance with equation (4.16).

When all the constants in equation (4.18) are known, including the specific resistance and dry cake mass per filtrate volume, possibly coming from laboratory test data, then filtration simulation can be performed:

filtrate volume in time t	$= V$	(m^3)
dry cake mass in time t	$= cV$	(kg)
wet cake mass in time t	$= m_R cV$	(kg)
mass slurry filtered in time t	$= m_R cV + \rho V$	(kg)

The height of the filter cake can be obtained from a volume balance conducted on the filter cake:

volume solids in cake + volume liquid in cake = cake volume

$$\frac{cV}{\rho_s} + \frac{(m_R - 1)cV}{\rho} = LA$$

which can be rearranged to give

$$L = \frac{cV}{A} \left(\frac{1}{\rho_s} + \frac{(m_R - 1)}{\rho} \right) \tag{4.22}$$

Industrial filtration equipment operating under constant pressure includes most vacuum filters, such as rotary vacuum (or drum) filters and rotary disc filters. One of these is illustrated in Figure 4.11.

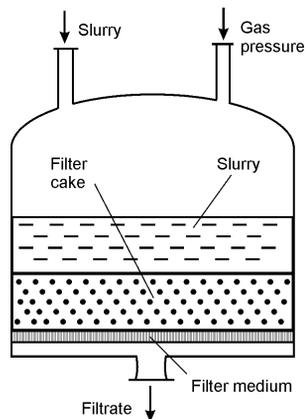


Fig. 4.13 Schematic diagram of a simple pressure Nutsche filter

Equation (4.11) can be solved under the condition of *constant rate filtration*. Under these conditions, the filtration pressure must increase in order to overcome the ever-increasing resistance to filtration due to cake growth. Equation (4.11) can be rearranged to give

$$\Delta P = \left(\frac{\mu \alpha c}{A^2} \frac{dV}{dt} \right) V + \frac{\mu R_m}{A \Delta P} \frac{dV}{dt} \tag{4.23}$$

which is the equation of a straight line on a plot of filtration pressure against filtrate volume. This is a result of the filtrate rate (dV/dt) being a constant – by definition. Hence, the gradient is provided by the terms within the brackets in equation (4.23) and the intercept is the furthest term on the right hand side of the equation.

Industrially, constant rate filtration occurs when a filter is fed by a positive displacement pump, that provides the same displacement of fluid with time; such as a piston pump, or progressing cavity pump. Thus a filter press fed by one of these pumps will operate under constant rate, dictated by the pump stroke or performance. However, it is usual for the mode of operation to switch from constant rate to constant pressure, when a set operating pressure has been achieved. This is a safety requirement to prevent over-pressurisation of seals, etc. An example of a filter press is provided in Figure 4.12 and a pressure Nutsche filter in Figure 4.13.

Feeding a filter press with a centrifugal pump will result in *variable rate and variable pressure filtration*. The pump will provide high flow at low back-pressure and this will fall to low flow at high pressure. Equation (4.11) has to be graphically, or numerically, solved rather than the analytical solutions represented by equations (4.18) and (4.23), see the box on the right. Firstly, using Q for the volume flow rate of slurry, and neglecting the volume of solids and liquid retained in the filter cake compared to the slurry volume, rearrangement of equation (4.11) provides

$$\frac{\Delta P}{Q} = \frac{\mu c \alpha}{A^2} V + \frac{\mu R_m}{A} \quad (4.24)$$

The pump characteristic provides a relation between the volume flow rate and the pressure drop and it is possible to rearrange and integrate the flow rate expression as follows

$$t = \int_0^V \frac{dV}{Q} \quad (4.25)$$

In this chapter we have considered bed filtration of suspensions with concentrations below 0.5 grams per litre and cake filtration at concentrations above 0.5% by volume. The question arises over what happens when concentrations less than 0.5% are used with conventional filtration media? The solids can block the surface pore openings, become lodged within the pore – blocking the flow, or can deposit within the pore thereby restricting the flow, but not blocking it. A general set of equations have been deduced to describe all of this behaviour

$$\frac{d^2 t}{dV^2} = K' \left(\frac{dt}{dV} \right)^{n'} \quad (4.26)$$

where K' and n' are constants. If a value of zero is used for n' , and the equation is integrated, then a similar equation to (4.18) results. Hence, this equation is a general one that includes cake filtration as one subset. Other values for n' are: 1, 1.5 and 2 and they represent

For more details on filtration equipment see www.midlandit.co.uk/particletechnology.

Variable pressure and rate filtration

The numerical solution is obtained by taking a pressure drop from the pump characteristic and the corresponding volume flow rate. These are used in equation (4.24) to calculate the filtrate volume, which is then plotted against the inverse volume flow rate as suggested by equation (4.25). Another pressure difference is then selected and the procedure repeated until a suitable plot of inverse flow rate against filtrate volume is produced. The area under the curve, up to a selected filtrate volume, provides the time taken to filter the required volume of filtrate, in accordance with equation (4.25).

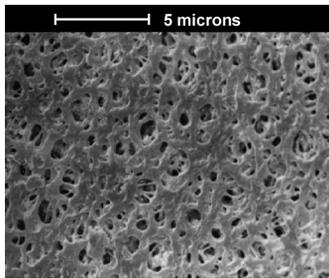


Fig. 4.14 Polymer microfiltration membrane – note scale bar

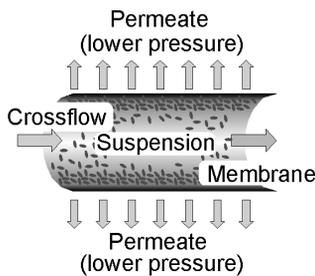


Fig. 4.15 Illustration of crossflow filtration – shear at the membrane surface helps reduce particle deposition.

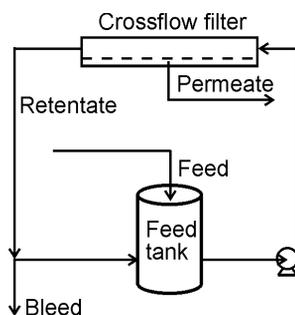


Fig. 4.16 Illustration of crossflow filtration process operation

intermediate blocking, standard blocking and complete blocking respectively. Standard blocking is used to describe the reduction in pore flow channel width due to particle deposition, whereas complete blocking is the closing off of the flow channel due to plugging.

4.6 Membrane filtration

In the context of particle technology, the most appropriate membrane filtration process is microfiltration (MF), but it is quite common to filter very fine particles using an ultrafiltration (UF) membrane. Also, UF mathematical models are sometimes applied to MF applications. So, both techniques will be briefly described here. A general definition of a MF membrane is one that possesses filtering pores between 10 and 0.1 μm . A UF membrane has pores below 0.1 μm , but is usually rated in terms of molecular weight cut-off, and is employed to retain macro-molecular material. Membranes usually possess a thin filtering surface, which may be supported by a much thicker structure. An example photograph, taken under a scanning electron microscope, is provided in Figure 4.14.

On the figure a variable pore structure is evident and the manufacturer's pore rating of the membrane is 0.2 μm , but surface openings up to 3 μm are evident. The filter is effective for the removal of all bacteria (sizes down to 0.2 μm), but it is obvious that these small particles will not be retained on the filter surface – at least some will penetrate the membrane matrix. The amount of penetration will depend upon the concentration of solids as, at high concentration, a cake may form. Thus, if all the fine particles are retained *at all concentrations* deposition must take place inside the filter and the microfilter is relying on depth filtration mechanisms similar to those described in Section 4.1. The consequence of internal deposition of particles within the membrane is that the filtration flux rate (or permeate rate) will decrease, if a constant pressure differential is being used. The permeate rate will also decrease as solids become deposited on the surface of the membrane, for similar reasons as described during cake filtration. In order to limit the occurrence of a cake *crossflow filtration* is often employed. This is illustrated in Figure 4.15.

The process selection of either crossflow operation or filtration in dead-end mode using, for example, a cartridge filter depends upon the process conditions. Cartridge membrane filters are used extensively in the electronics and medical industries to ensure sterility and particle free fluids (gases as well as liquids). The industry is worth well over \$1 billion per year. In most of these applications the concentration of suspended material is very small and the filter operates in dead-end and it is discarded when the pressure drop required to maintain an acceptable flow rate becomes too great. However, these are high value industries that can afford to discard these products. A duplex system is often used: a second

cartridge filter is used whilst the first is taken off-line for filter element changing. However, at higher concentrations of suspended solids the frequency of element changing would be too great and crossflow filtration may be appropriate. In this instance a clean liquid stream is still supplied, but a second liquid stream containing a higher concentration of suspended material is recycled. This stream is the *retentate* and the process is illustrated in Figure 4.16.

Despite the application of crossflow filtration the permeate flux rate usually decreases due to the deposition of some solids on the surface of the filter, as well as the deposition within filters when using the design illustrated in Figure 4.14. A typical flux rate curve is illustrated in Figure 4.17. The conventional units of membrane flux are litres of permeate per square metre of filter per hour ($l\ m^{-2}\ h^{-1}$). It may be possible to momentarily *backflush* some permeate through the membrane to dislodge the solids retained on the surface of the membrane, which will then be swept away in the crossflow, thus restoring some of the original flux. However, it is much more difficult to remove the solids deposited within the membrane filter matrix and this may lead to an ever-declining flux rate, even with backflushing applied.

During crossflow filtration two stages of modelling may be considered: the period in which the flux declines, and a model to predict the equilibrium value of the filtration flux. Flux decline models are often based on pore blocking, or diminishing size, concepts discussed in Section 4.5 and numerically the data is fitted to versions of equation (4.26). From a practical perspective, models of the equilibrium flux rate have greater application because these enable the calculation of flow and process conditions required to perform an industrial separation over an extended period in time. Two general models are often used: cake filtration and film theory. These are illustrated in Figure 4.18. Film theory is normally applied to very small particles, less than $5\ \mu m$, and is founded in classical mass transfer analysis. Cake filtration is usually applied to larger particles and some of the resistance terms are described by the equations discussed in Section 4.5, with additional forces due to colloidal interactions as described in Chapter 13.

In MF the concentration of suspended material passing through the membrane is usually negligible, $c_p=0$, hence the *retention* (R_t) is 100%

$$R_t = \left(1 - \frac{c_p}{c_b}\right) 100\% \quad (4.27)$$

In the cake filtration model the permeate flux rate (J) is described by a form of the cake filtration equation (4.7)

$$J = \frac{\Delta P_{TMP}}{\mu(R_m + R_c - R_s)} \quad (4.28)$$

where the flux is

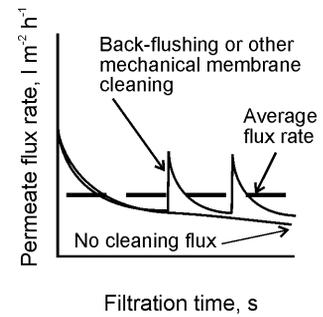


Fig. 4.17 Permeate flux rate behaviour with backflushing to restore the rate

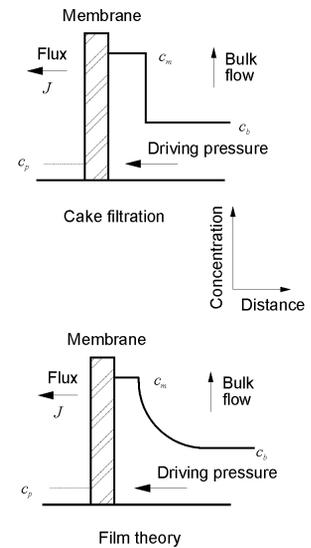
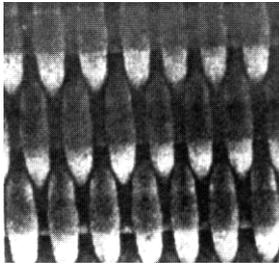
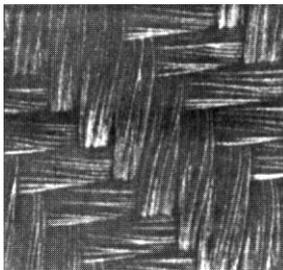


Fig. 4.18 Cake and film models of membrane filtration

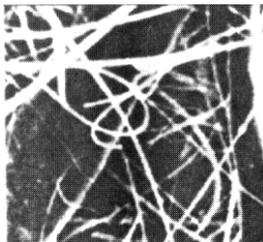
$$J = \frac{dV}{dt} \frac{1}{A} \quad (4.29)$$



monofilament



multifilament



needle felt

Fig. 4.19 Examples of filter cloths

and the filtration is run under the conditions of constant pressure. The filtration resistance terms are: membrane, cake and shear (for crossflow). The latter term is negative because the crossflow shear causes a reduction in the resistance. In many cases it is possible to relate the reduction in resistance due to shear with the flow condition, such as surface shear stress, flow rate, degree of turbulence, etc.

In the film model, the convective transport of solids towards the membrane is balanced, at equilibrium, by the diffusional transport away from the membrane. This gives rise to a smooth concentration gradient approaching the membrane, rather than the step change seen in cake filtration. This gradient in concentration of material is called *concentration polarisation* and, in the film model, it is here that the resistance to filtration is assumed to be greatest. Thus, a film equation is appropriate

$$J = \left(\frac{D_e}{\delta} \right) \ln \left(\frac{c_m - c_p}{c_b - c_p} \right) \quad (4.30)$$

where c_m is the concentration at the membrane surface, D_e is the effective diffusivity of the filtering species and δ is the distance over which the diffusion takes place. The last two terms can be replaced by a mass transfer coefficient, as is conventional in film modelling, and that coefficient can be related to flow conditions using film transfer correlations. This enables the prediction of permeate flux under different conditions of shear, crossflow velocity, etc. The concentration c_m is assumed to be a constant, once sufficient material has arrived at the membrane to form it, so that an increase in filtering pressure will not give rise to an increase in c_m . Hence, once the value of c_m is achieved at the wall, the flux rate determined by equation (4.30) will not be influenced by further increases in pressure because the mass transfer resistance is within the diffusional region represented by the concentration polarisation. Thus, permeate flux will be transmembrane pressure independent, after the threshold of reaching c_m at the membrane. Increasing the pressure difference will increase the thickness of the c_m deposit, but will not influence the concentration gradient at the membrane wall, equation (4.30).

4.7 Filter media

In the last section we met particle deposition within the filtration medium, resulting in long-term flux decline; i.e. increasing filtration resistance. In the cake filtration section we discussed the need for bridging over the pores within the filter medium, see Figure 4.5. The filtration medium used for conventional cake filtration has a much more open structure than a membrane and is more mechanically robust. Usually, woven cloths are employed, or pressed needle felts; these are illustrated in Figure 4.19. Pores within the cloth are much

bigger than the particles and this helps minimise the long-term increase in medium resistance. However, the situation described by Figure 4.5 is too simplistic, a more realistic representation is provided in Figure 4.20: it is difficult to define where the cake finishes and the filter medium starts and there is some penetration of the medium by the particles. One important consequence of this is that it is not possible for filter cloth manufacturers to define a value for R_m for their cloth. The value must be determined *in-situ*, or during laboratory tests on the slurry, as it is a function of both the cloth and the material being filtered. Typically, the *in-situ* medium resistance is one or two orders of magnitude greater than the clean value. In many cases this is also true during membrane as well as cake filtration.

One of the main distinctions between filtration media for cake filtration is the use of monofilament or multifilament fibres for the weaved cloth. Monofilaments give better cake discharge properties, lower resistance due to internal solids deposition and lower flow resistance in general; however, they are less effective at particle retention and can be weaker cloths. Plugging of a monofilament is also possible; whereas flow is possible between the fibres constituting the multifilament cloth, so complete plugging is unlikely. When filtering fine particles both types of cloths may not provide a good clarity filtrate from the start and these *first runnings* can be recycled back into the feed stream until the clarity of the filtrate improves.

Factors to be considered when selecting a filter medium, apart from the ability to retain particles or form a cake, include: good cake release, chemical compatibility with the liquid and solid to be filtered, mechanical ability to withstand the filtration cycle (often including stretching and rolling), cost, economic filtration time, adequate cloth lifetime and resistance to particle blinding and biological growth. In operation, it is common to include a cloth washing stage between cake discharge and cloth reuse, to enhance cloth performance.

Laboratory tests to determine the cake resistance, as mentioned in Section 4.5, are also used to evaluate different types of filter cloth and their resistances, for possible use on a production filter. However, as the interaction between the cloth and solids is significant, see Figure 4.20, it is important that the filtration test is performed following the same orientation that the production filter will use. A rotary vacuum filter has a filtering surface facing downwards; thus coarser and easier to filter particles will settle away from the surface, so the test should also use a downward facing filter leaf. An example filter leaf is illustrated in Figure 4.21.

When filtering finely divided material that may *blind* a cloth, or filter membrane, it is possible to deposit a sacrificial initial layer of solids of an additional material. The filtration takes place on the deposited solids and, when the filtration performance becomes unacceptable, the layer can be removed and replenished with a fresh one. This is called *pre-coat* filtration and secondary membrane filtration in classic and membrane filtrations, respectively.

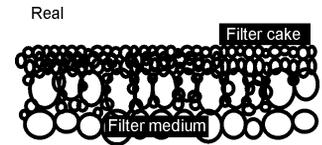


Fig. 4.20 More realistic representation of particles on a filter cloth – c.f. Figure 4.5

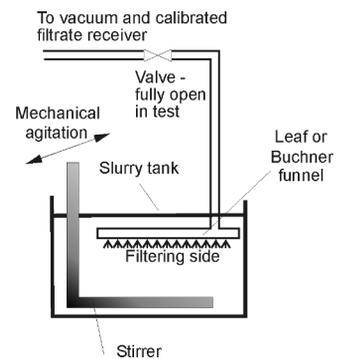


Fig. 4.21 Laboratory test filter leaf – usually at constant pressure and the filter orientation should be similar to how the filter will be used

4.8 Filter aids

Pre-coat filtration uses a filter aid to protect the filter medium and the filter aid might also be added to the suspension to be filtered in what is called a *body feed*. Hence, it may be possible to filter very finely divided solids using the filter aid to capture the particles by similar mechanisms to bed filtration described earlier, but employing conventional cake filtration equipment. This is only economic for the removal of low concentrations of finely suspended solids because the filter aid is rarely regenerated.

Two commonly encountered filter aids are diatomaceous earth (kieselguhr) and perlite. The former is the skeletal remains of diatoms, a type of algae, that died thousands of years ago and compacted into sedimentary deposits that can be mined. The latter is a glassy volcanic mineral that expands on heating and has usually been crushed. Both types provide highly porous particle beds due to their very irregular particle shapes. The dendritic, or spiky, nature of the kieselguhr prevents two particles approaching closely and provides an open flow path between the spikes. However, capture of fine particles on the spikes is still possible.

Recently, the trend has been away from conventional filter aids because it may be difficult to dispose of the silica based ones. When the cake dries out there is a possibility of the finely divided silica particles posing a health problem. This has increased disposal costs and led to the adoption of alternatives such as fuel ash, sawdust and other waste solid products as filter aids.

4.9 Summary

In this chapter, we have extended the principles covered in Chapter 3 to cover the effects of depositing solids, within a packed bed and on top of a filter medium. In most cases we are concerned with the pressure drop required for a given flow rate of filtrate, or permeate. This is required for equipment design, or simply to understand the process. Numerous filter types are available, these are not described in detail, but the hydraulic operating principles are. Filter cake washing and air blowing, to displace moisture from the cake prior to thermal drying, are not covered but will be considered in Section 8.5.

The reader should appreciate the many similarities between conventional and membrane filtrations, including the tendency for the filter medium resistance to be a function of what is being filtered and not just the medium itself. For most filter mediums it is not possible to define a true pore size, or diameter, but an *equivalent* one provided by a flow test based on hydraulic resistance, or the ability to displace one fluid. This is met again in Section 13.5 and is called the *bubble point* test.

Selection of equipment for solid/liquid separation depends on many factors, both equipment and process related. Expert systems exist for this purpose and the reader is referred to one that is freely available on the left. However, some existing operating data, or laboratory tests, are essential for reliable equipment specification.

Equipment selection

Attempts to specify equipment based simply on particle size are not successful. However, in general finer particles do require higher operating pressures for filtration. A free expert system for selection is available at: www.filtration-and-separation.com

4.10 Problems

1.

A 3 layer filter is used to clarify an effluent containing 60 mg litre⁻¹ (ppm) of solids. Complete the following table:

Depth (m)	Bed medium	Initial filtration constant (m ⁻¹)	Inlet concentration to section (ppm)	Outlet concentration from section (ppm)
0.45	anthracite	2	60	
0.28	sand	8		
0.22	alumina	15		

The final treated effluent concentration discharged from the filter will be (ppm):.....

2. Five litres of a 10% w/w slurry of chalk (ρ_s of 2670 kg m⁻³) in water (μ of 0.001 Pa s) was filtered under *constant pressure* on a filter leaf of 0.0314 m² area. The specific surface of the chalk is 3x10⁶ m⁻¹ and the cake can be assumed to have had a porosity of 50%.

- i). The volume of solids in the slurry was (cm³):
a: 100 b: 500 c: 200 d: 250
- ii). The volume of water retained in the filter cake was (cm³):
a: 400 b: 200 c: 100 d: 0
- iii). The filtrate volume was (cm³):
a: 4600 b: 4750 c: 4500 d: 4800
- iv). The mass of dry solids in the cake was (kg);
a: 0.533 b: 0.500 c: 1.335 d: 0.668
- v). The dry mass of solids per unit volume of filtrate (i.e. the same as 'c') was (kg m⁻³):
a: 112 b: 100 c: 116 d: 2670
- vi). Using the Kozeny-Carman equation, the permeability of the cake was (m²):
a: 1.1x10⁻¹⁴ b: 9x10¹³ c: 3.3x10⁻⁸ d: 3x10⁷
- vii). The specific resistance of the cake was (m kg⁻¹):
a: 8.3x10⁻¹⁸ b: 22500 c: 2.5x10⁻¹¹ d: 6.7x10¹⁰
- viii). The time taken to filter the suspension, if the medium resistance can be neglected and the differential pressure forming the cake was 36 cm of mercury (ρ_{Hg} =13600 kg m⁻³), was (s):
a: 55 b: 998 c: 1572 d: 1747

3. The same quantity of a similar suspension to problem 2 was filtered on the same filter at *constant rate* until 2.3 litres of filtrate had been recovered. The following empirical equation for pressure was observed

$$\Delta P = 168\text{Pa/second} + 6670\text{Pa}$$

Deep bed filtration

The following differential equation describes the distribution of solids within the filter bed:

$$\left(\frac{\partial N}{\partial z}\right)_t = -\lambda N$$

where N is solid concentration in any consistent set of units and λ is the filtration constant. By considering only the start of the filtration the above partial differential equation may be treated as an ordinary differential equation, and integrated with respect to position (z)

Cake filtration

Resistances in series are additive

$$\Delta P_{\text{total}} = \Delta P_{\text{cake}} + \Delta P_{\text{medium}}$$

Hence

$$\Delta P = \frac{\mu \alpha c}{A^2} V \frac{dV}{dt} + \frac{\mu R_m}{A} \frac{dV}{dt}$$

where c is the dry mass of cake deposited per unit volume of filtrate:

$$c = \frac{s\rho}{(1 - m_R s)}$$

where s is slurry concentration as a weight fraction and m_R is mass wet cake sample/dried sample (i.e. the moisture ratio). The connection between specific resistance (α) and permeability (k) is

$$\alpha = \frac{1}{k(1 - \epsilon)\rho_s}$$

Assume an incompressible filtration: i.e. α , c as well as A and μ were unchanged from above.

i). The pressure drop over the medium during the filtration was (Pa):

- a: need R_m b: $168t+6670$ c: 6670 d: need dV/dt

ii). The constant filtrate rate was ($m^3 s^{-1}$) - hint compare cake filtration equation and the empirical equation for pressure rise:

- a: 3.4×10^{-6} b: 3.7×10^{-7} c: 9.7×10^{-5} d: 4.6×10^{-6}

iii). The time taken to achieve the filtration to this stage was (s):

- a: 678 b: 6287 c: 24 d: 500

iv). The medium resistance was (m^{-1}) - hint compare the equations again - consider the two terms separately, this term is static:

- a: 6.2×10^{10} b: 5.7×10^{11} c: 2.2×10^9 d: 4.5×10^{10}

Effective medium resistance

A previously formed cake can be treated as being part of an effective medium resistance (R_m'), and the equation for the next filtration can then be applied to a second half of the filtration. The effective medium resistance has two components: the true medium resistance and that due to the earlier cake formation

$$R_m' = R_m + c\alpha \frac{V}{A}$$

4. Following on from problem 3, after the 2.3 litres of filtrate was recovered the pressure was then kept constant until all the suspension was filtered. Assume an incompressible filtration: i.e. α , c , A and μ were the same as in Q.1 and see the box on effective medium resistance.

i). The effective medium resistance was (m^{-1}):

- a: 4.5×10^{10} b: 5.7×10^{11} c: 6.2×10^{11} d: 1.2×10^{12}

ii). The volume of filtrate filtered under constant pressure was (m^3):

- a: 0 b: 0.0023 c: 0.0046 d: 0.0025

iii). The total filtration pressure was (Pa):

- a: 90600 b: 10650 c: 1065000 d: 121000

iv). The time filtering at constant pressure was (s):

- a: 519 b: 674 c: 731 d: 2470

v). The total filtration time was (s):

- a: 1230 b: 1350 c: 2490 d: 6810

5. A pilot experiment using a filter of $0.1 m^2$ area at a constant pressure of $68.5 \times 10^4 Pa$ produced the results on the left. The filtrate viscosity was $0.0015 Pa s$, slurry concentration was 3% w/w, cake concentration was 52% w/w and the liquid density was $1000 kg m^{-3}$.

Filt'n time (mins)	Filtrate volume (litres)	Time over volume ($s m^{-3}$):
10	88	
20	125	
40	180	
60	220	

i). The moisture ratio was - hint consider 100 kg of wet cake:

- a: 0.52 b: 1.92 c: 1 d: 0.30

ii). The dry solids per unit volume of filtrate was ($kg m^{-3}$):

- a: 30.9 b: 1080 c: 17.3 d: 31.8

iii). Draw a graph and calculate: the specific resistance of the cake and the resistance of the medium

iv). On a $10 m^2$ filter what would be the filtrate volume after 2 hours?

v). If the solid density is $2500 kg m^{-3}$, what would be the cake thickness in part (iv)?