

3 Fluid flow in porous media

In Chapter 2 we considered how to represent a particle size distribution by, where possible, a single term that is representative of all the particle sizes. This term may then be used for modelling, design or simply to understand a process within Particle Technology. One such example is in the fluid flow through a porous medium, or porous media (plural). There are a number of practical applications of fluid flow, including filtration, flow in a packed column, permeation of water, or oil, within the matrix of a porous rock, etc. Before discussing the consequences of our choice of a single value to represent the distribution, and the appropriate modelling equations, we must define the commonly used terms.

3.1 Definitions

By definition, a porous medium consists of pores between some particulate phase, contained within a vessel, or some control volume, as illustrated in Figure 3.1. The fluid flow rate through the bed is Q ($\text{m}^3 \text{s}^{-1}$) and the bed cross sectional area is A (m^2). Thus the superficial (or empty tube) velocity U_o is the total flow rate divided by the cross sectional area. The existence of the particles within the bed will reduce the area available for fluid flow; i.e. to preserve fluid continuity with the entering superficial flow the fluid will have to squeeze through a smaller area; hence the velocity *within* the bed (U – interstitial velocity) will be greater than the superficial. In Particle Technology calculations it is the volume fraction that is most important, and not the mass fraction. The volume fraction of solids present (i.e. volume solids in bed divided by total bed volume) is usually referred to simply as the *volume concentration*, or solids fraction, and the remaining fraction is that of the voids. The void fraction is also called the *voidage* and the bed *porosity*. It is important to realise that, in liquid systems, the voids are usually filled with liquid and not to assume that the bed consists of just solids and air. The porosity is usually an isotropic property (i.e. the same in all directions); hence, the interstitial velocity is simply related to the superficial velocity by the following expression, which comes from a consideration of fluid continuity.

$$U = \frac{U_o}{\epsilon} \tag{3.1}$$

Clearly, the resistance to fluid flow through the porous medium is related to the amount of particles present, or volume concentration, but it is conventional to work in terms of bed porosity. At one extreme, when the bed is full of solids (porosity is zero – possible with cubic particles placed carefully within the bed) the resistance is infinite. At the other, when no solids are present and the porosity is unity, the interstitial velocity will be the same as the superficial velocity. The resistance to fluid flow gives rise to a pressure drop in

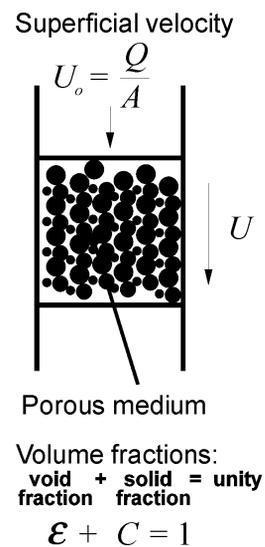


Fig. 3.1 Illustration of fluid flow through a porous medium and consideration of the volume fractions present

exercise 3.1
 Using continuity:
 i.e. $Q = \text{constant}$,
 deduce equation (3.1)

Measuring porosity

For a dry powder in air: fill a weighed measuring cylinder to a graduation, gently vibrate and reweigh. The bed mass over volume will give the *bulk density* (ρ_b) of the powder:

$$\rho_b = (1 - \varepsilon)\rho_s + \varepsilon\rho$$

As the fluid is a gas, its contribution to the bulk density is minimal and

$$\varepsilon = 1 - \rho_b / \rho_s$$

the fluid (ΔP). Pressure is not a vector quantity, but a pressure gradient with respect to distance ($\Delta P/L$) is. The pressure decreases in the direction of the fluid velocity, hence the pressure gradient should be negative: ($-\Delta P/L$). However, for the sake of brevity, we will adopt the term *pressure difference*, which is a scalar quantity, hence the negative symbol will not be used in the following text.

The porosity of a packed bed of material depends strongly upon the nature of the particles and how the bed has been treated. Incompressible glass beads, including marbles, pack to a porosity of about 45%, but beds of alumina particles (not catalyst pellets) often reach porosities of 75%. Biological material, such as yeast cells, may form a packed bed with porosities of 90%, or higher. Therefore, it is dangerous to make the common assumption that packed beds are 50% solids and 50% voids. It is a simple parameter to measure, so long as the solid and fluid densities are known; see the box on the left.

3.2 Flow regimes

On page 1 the flow Reynolds number was stated. In a porous medium, as with all fluid flow problems, we need to consider energy losses from the fluid due to viscous and form drags. The former can be simply referred to as laminar flow (low Re) whereas turbulent flow has additional drag due to eddies in the fluid within the porous medium. The *Modified Reynolds* number is used to determine the flow regime of the fluid within the porous medium. Modification to the fluid velocity term (u) and the characteristic linear dimension (d) are required. When considering flow within the bed the appropriate velocity is the interstitial, hence $u = U$, which can be related to the superficial velocity by equation (3.1). The characteristic linear dimension was deduced by Kozeny and is the volume open to the fluid flow divided by the surface area over which it must flow (i.e. product of volume of solids and specific surface area per unit volume)

$$d = \frac{AL\varepsilon}{AL(1-\varepsilon)S_v} = \frac{\varepsilon}{(1-\varepsilon)S_v} \quad (3.2)$$

Thus, equations (3.1) and (3.2) used in the Reynolds number expression give the Modified Reynolds number (Re_1)

$$Re_1 = \frac{\varepsilon}{(1-\varepsilon)S_v} \frac{U_o \rho}{\varepsilon \mu} = \frac{\rho U_o}{(1-\varepsilon)S_v \mu} \quad (3.3)$$

Conceptually, the number still represents the ratio of inertial to viscous forces in the fluid and provides a means to assess when the inertial effects become significant. The conventionally applied threshold to indicate significant turbulence is 2, whereas for the flow Reynolds number (page 1) the conventional threshold is about 2000. It is important to note that the density term in equation (3.3) is the density of the fluid: the turbulences described are that of the fluid, the particles do not move in a packed bed.

3.3 Darcy's law and the Kozeny-Carman equation

Darcy's law, and the Kozeny-Carman equation, are valid for laminar flow ($Re_1 < 2$); i.e. for viscous drag by the fluid on the surface of the particles within the bed. The analogy with electrical flow is shown in Figure 3.2. A cell, or pump, provides the driving potential and the flow, in either system, depends on the resistances in the circuit. For two equal resistances, the potential, or pressure, is equally divided between the two. In fluid flow, high resistance is provided by high fluid viscosity (treacle is more difficult to pump than air) and by low permeability (k) of the bed. A value of zero permeability would give rise to infinite resistance - both for electrical and fluid flow. Darcy's law¹ is

$$\frac{\Delta P}{L} = \frac{\mu}{k} \frac{dV}{dt} \frac{1}{A} \tag{3.4}$$

where V is the volume of fluid flowing in time t . For a given bed length the pressure drop will rise linearly with volume flow rate, or fluid velocity, noting that

$$U_o = \frac{Q}{A} = \frac{dV}{dt} \frac{1}{A} \tag{3.5}$$

as illustrated in Figure 3.3. The permeability of a packed bed can be measured in this way, using the gradient from such a plot. The permeability is often assumed to be a constant in a packed bed, provided the particle packing is also uniform within the bed. It should, therefore, be an intrinsic property of a material. However, in order to use equation (3.4) for design, e.g. to specify a pump required to pass liquid through a bed at a desired flow rate, we need a method for predicting the permeability of the bed. This is provided by the Kozeny-Carman equation.

The Kozeny-Carman equation was derived from the Hagen-Poiseuille equation for laminar flow of a fluid in a circular channel

$$\frac{\Delta P}{L} = \frac{32\mu}{d^2} u \tag{3.6}$$

where d is the channel diameter. The derivation assumed that flow in a porous medium can be represented as flow through many parallel channels and equations (3.1) and (3.2) were used to represent the equivalent channel diameter and to convert between the fluid velocity within the channel and the superficial. Hence, substituting these equations in (3.6) and collecting the constants together in a single term called *The Kozeny constant (K)*, which includes a factor relating the tortuous flow channel length to the measured bed depth gives the Kozeny-Carman equation

$$\frac{\Delta P}{L} = \mu \left[\frac{K(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] U_o = \mu \left[\frac{K(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] \frac{dV}{dt} \frac{1}{A} \tag{3.7}$$

¹ Darcy observed the law during the 1850's by monitoring pressure drop over sand filters at Dijon, France.

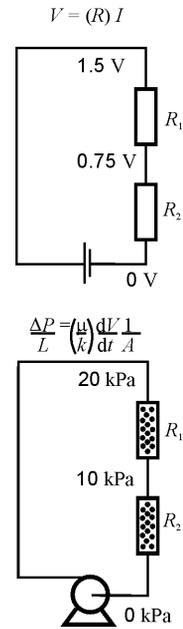


Fig. 3.2 Analogy between fluid and electrical flows. The flow rate (current or fluid) is proportional to the driving potential: voltage in Ohms law and pressure gradient in a fluid. The constant of proportionality is resistance (R) - which for a fluid is viscosity divided by bed permeability (k).

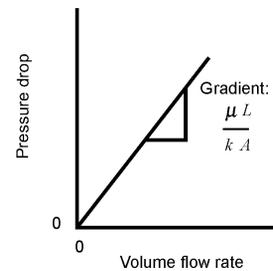


Fig. 3.3 Graphical representation of Darcy's law for a bed of fixed overall length

Permeability

Comparing equations (3.4) and (3.7) the permeability is

$$k = \frac{\epsilon^3}{K(1-\epsilon)^2 S_v^2}$$

hence the SI unit is m².

The Kozeny constant is often 5, but there is much experimental evidence to suggest that

$$K = f(\epsilon)$$

i.e. the Kozeny coefficient is a function of porosity.

Comparison of equations (3.4) and (3.7), results in the conclusion that the Kozeny-Carman equation is simply a subset of Darcy’s law, with an analytical expression for permeability. There are many alternative expressions for permeability, but the Kozeny approach is the most frequently encountered. In many instances the Kozeny constant has a value close to 5, but this is not universally true. Inspection of equation (3.7) shows that the permeability, or inverse resistance to fluid flow, is dependent upon the bed porosity (or solids concentration) and the specific surface area per unit volume of the particles within the bed. This is logical because the higher the bed surface area the greater the viscous drag of the fluid on the particles. Equation (2.14) can be substituted into (3.7) to provide a version of the equation in terms of particle size

$$\frac{\Delta P}{L} = \mu \left[\frac{36K(1-\epsilon)^2}{\epsilon^3 x_{sv}^2} \right] U_o \tag{3.8}$$

where x_{sv} is the Sauter mean diameter for the particle distribution. On page 6 and 7, the question ‘which particle diameter to use to represent the distribution?’ was asked and Figure 2.2 provided an example distribution. The x_{50} and x_{sv} for this distribution are 9.2 and 6.4 μm , respectively. The consequence of using an inappropriate equivalent spherical diameter can be illustrated by using both of these diameters in equation (3.8) and comparing the results. Under identical flow conditions, the ratio of pressure drops calculated by the Sauter mean to that by the median particle size is 2:1; i.e. there is 100% difference between the pressure drops calculated by these diameters; yet both are equivalent spherical diameters for the same particle size distribution. Clearly, the Sauter mean is the most appropriate diameter to use, as indicated in equation (3.8), but it is a calculated value and the median is quick to read off the cumulative distribution curve. Hence, there is great temptation to use the median, but it would predict only 50% of the likely pressure drop.

3.4 Friction factor

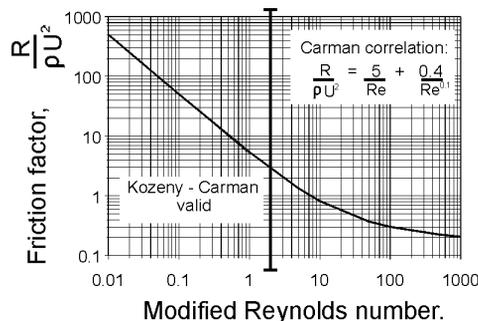


Fig. 3.4 The friction factor plot for fluid flow through porous media

When turbulences within the fluid flowing through the porous medium become significant, i.e. Modified Reynolds numbers greater than 2, additional drag terms to the viscous ones quantified in the last section become important. In fluid flow through pipes and channels a friction factor was deduced to represent this region and Carman extended the analogy with pipe flow to cover both flow regions in porous media. The porous media friction factor is illustrated in Figure 3.4 and the method used to relate the shear stress at the surface of the solids, to the pressure drop, follows (the same approach for flow in pipes is included in the box overleaf for comparison).

$$\frac{R}{\rho U^2} \tag{3.9}$$

Equation (3.9) is the friction factor and R is the shear stress, or drag force per unit area, on the particle surface. A force balance at the particle surface can be constructed as follows.

$$\text{surface area of particles} = S_v(1-\varepsilon)LA \tag{m^2}$$

$$\text{drag force} = R \cdot \text{particle surface area} \tag{N}$$

and

$$\text{pressure drop on fluid} = \Delta P \tag{N m^{-2}}$$

$$\text{force by fluid} = \Delta P A \varepsilon \tag{N}$$

Equating the two forces and rearranging gives

$$R = \frac{\varepsilon}{S_v(1-\varepsilon)} \frac{\Delta P}{L} \tag{3.10}$$

Note that equation (3.10) is the analogue of that provided for pipe flow in the box. Finally, expanding into a friction factor, equation (3.9), together with equation (3.1) gives

$$\frac{R}{\rho U^2} = \frac{\varepsilon^3}{S_v(1-\varepsilon)} \frac{\Delta P}{L} \frac{1}{\rho U_o^2} \tag{3.11}$$

Alternatively, the pressure drop per unit length is

$$\frac{\Delta P}{L} = \left(\frac{R}{\rho U^2} \right) \frac{S_v(1-\varepsilon)\rho U_o^2}{\varepsilon^3} \tag{3.12}$$

where the bracketed term is the friction factor. So, given a flow rate, hence superficial velocity, it is possible to calculate the Modified Reynolds number from equation (3.3) and the friction factor from Figure 3.4. This can then be used in equation (3.12) to provide the pressure drop, or gradient, under conditions of laminar or turbulent flow through the porous medium.

3.5 Carman and Ergun correlations

The friction factor plot, with Reynolds number, for fluid flow through porous media is a smoother function than that found in pipe flow. This is due to the smooth increase in turbulences within the bed as flow rate increases. Thus the friction factor plot can be represented by just one, or two, empirical curves. The Carman correlation is generally used for solid objects forming a bed

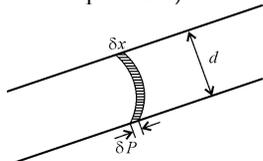
$$\frac{R}{\rho U^2} = \frac{5}{Re_1} + \frac{0.4}{Re_1^{0.1}} \tag{3.13}$$

The Ergun correlation is for hollow objects, such as packing rings

$$\frac{R}{\rho U^2} = \frac{4.17}{Re_1} + 0.29 \tag{3.14}$$

The form of both equations is similar: with a correction term added to the laminar flow term to account for resistance due to turbulences.

Force balance on a pipe wall
(for pure fluid - no particles)



force on wall:
 $R \pi d \cdot \delta x$

force on fluid:
 $\delta P \frac{\pi d^2}{4}$

combine and integrate:
 $RL = \frac{\Delta P d}{4}$ or
 $R = \frac{d \Delta P}{4 L}$
c.f. equation (3.10)

Laminar flow
Carman correlation without the turbulent correction is

$$\frac{R}{\rho U^2} = \frac{5}{Re_1}$$

Using equation (3.10) and (3.3) provides

$$\frac{\varepsilon^3}{S_v(1-\varepsilon)} \frac{\Delta P}{L} \frac{1}{\rho U_o^2} = \dots$$

$$\dots = \frac{K(1-\varepsilon)S_v\mu}{\rho U_o}$$

So,

$$\frac{\Delta P}{L} = \mu \left[\frac{K(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] U_o$$

i.e. equation (3.7)

Packing arrangements
Uniform spheres, packed together in a regular pattern, vary from a coordination number of 6, for simple cubic packing, to 12 for hexagonal close-packed; which is the closest possible packing for uniform spheres. The solids concentration for these arrangements varies from 0.524 to 0.740, respectively (porosities of 0.476 to 0.260). Randomly packed spheres have a solid concentration of 0.50 to 0.60. In theory, with a size distributed solids the finer particles could fit inside the gaps between the larger particles providing even higher solid packing. In practice, solid concentrations much lower than the regular packed arrays are found. It is always safest to measure concentration, or porosity, as described on page 22.

This is illustrated in the box, which shows that the Carman correlation reduces to the Kozeny-Carman equation, with $K=5$, when the turbulent correction term is dropped.

In practice, when performing flow calculations with Modified Reynolds numbers greater than 2, equations (3.13) or (3.14) are used to determine the friction factor – rather than Figure 3.4, and equation (3.12) is used to calculate the pressure drop.

3.6 Concentrations by mass and volume

Solid concentration by volume fraction (C) was illustrated under Figure 3.1 and is simply the volume of solids present divided by the total bed volume. It is numerically equal to unity minus bed porosity. In most of the following chapters, it is more convenient to work in terms of solid concentration rather than porosity. However, laboratory analyses often provide solid concentration by mass. For example, taking a sample of a filter cake containing water: weighing, drying and then weighing the dried cake will provide the concentration by mass – if the last mass is divided by the first. Hence, conversion between the two different types of concentrations is frequently required.

Consideration of what the solid concentration by volume fraction means leads to the following expression

$$C = \frac{\text{volume solids}}{\text{volume solids} + \text{volume fluid}}$$

If a sample has resulted in a concentration by mass (C_w), and the total sample mass is M , then the volumes present can be deduced if the densities of the solid and fluid are known by

$$C = \frac{C_w M / \rho_s}{C_w M / \rho_s + (1 - C_w) M / \rho} \quad (3.15)$$

Dividing through by the volume of solids gives

$$C = \frac{1}{1 + \frac{(1 - C_w) \rho_s}{C_w \rho}} \quad (3.16)$$

A similar argument can be applied to convert from concentration by volume fraction to mass fraction.

3.7 Summary

In this chapter we have seen the importance of specific surface: it defines the surface area that is present within a porous medium; and it is the friction of fluid flowing over that area that causes a pressure drop. Finer particles provide a higher surface area per unit volume than coarser ones, therefore, a higher flow resistance. Turbulences within the fluid inside the bed, at higher flow rates, cause an additional flow resistance, or pressure drop. Thus, a calculated pressure drop by a laminar flow equation, Darcy's law or Kozeny-Carman, will always underestimate the true pressure drop if significant turbulence is present. Often the temptation to use Kozeny-

Carman, rather than the procedure described in Sections 3.4 and 3.5, is too great even when the Modified Reynolds number is high. This will lead to an under-design, or specification, for equipment such as pumps and fans. Providing another example of the failure in design discussed in the Forward.

Even under conditions of laminar flow, the use of the median size to represent the size distributed solids rather than the Sauter mean diameter can cause significant errors, as shown in Section 3.3. Finally, a brief consideration of particle packing arrangements has been included, but in most processes the packing arrangement is random and not structured. Hence, the safest procedure for the analysis of a flow through porous media problem is to conduct experiments to deduce characteristics such as the permeability and if it varies with flow; a possibility if the finer particles become transported within the bed or if suspended solids within the fluid deposit inside the bed. The latter is depth filtration, which is covered in the next chapter.

3.8 Problems

1.

(i) A powder is contained in a vessel to form a cylindrical plug 0.8 cm in diameter and 3 cm long. The powder density is 2.5 g cm^{-3} and 2.20 grams of powder was used to form the plug. The porosity inside the plug of powder is (-):

a: 0.58 b: 0.42 c: 0.75 d: 0.25

(ii). Air was drawn through the plug at a rate of 6.6 cm^3 per minute. A mercury manometer was used to measure the pressure drop during this process: a pressure drop of 60 mm Hg was recorded. The specific gravity of mercury is 13.6, thus the pressure drop across the plug was (Pa):

a: 80 b: 800 c: 8000 d: 80000

(iii). The superficial gas velocity in (ii) was (m s^{-1}):

a: 0.0022 b: 3.65×10^{-5} c: 2.2×10^{-6} d: 0.00365

(iv). The viscosity of the air was $1.8 \times 10^{-5} \text{ Pa s}$, using the Kozeny-Carman equation, the specific surface area per unit volume of the powder was (m^{-1}):

a: 2310 b: 3.0×10^{11} c: 5.5×10^5 d: 1.2×10^6

(v). The Sauter mean diameter of the powder was (μm):

a: 2600 b: 22 c: 11 d: 5.0

(vi). The air density was 1.2 kg m^{-3} , the Modified Reynolds Number of the system was (-):

a: 0.10 b: 8.4×10^{-10} c: 4.6×10^{-4} d: 0.00021

Equation summary

Under laminar flow:

$$\frac{\Delta P}{L} = \mu \left[\frac{K(1-\varepsilon)^2 S_v^2}{\varepsilon^3} \right] U_o$$

where K is the Kozeny constant. At high values of the 'Modified Reynolds Number' ($\text{Re}_1 > 2$):

$$\text{Re}_1 = \frac{U_o \rho}{\mu(1-\varepsilon)S_v}$$

turbulent conditions pertain. A pressure drop given a flow rate can still be deduced, but first the Modified Reynolds number is required, then the friction factor using say the Carman correlation:

$$\frac{R}{\rho U^2} = \frac{5}{\text{Re}_1} + \frac{0.4}{\text{Re}_1^{0.1}}$$

A force balance on the surface of the solids and on the fluid gives:

$$RS_v LA(1-\varepsilon) = \Delta PA\varepsilon$$

where L is the bed height or depth. Given a value for the shear stress on the solids (R) calculated from the Carman correlation then the pressure drop (ΔP) can be calculated from the force balance.

(vii). Comment on whether your use of the Kozeny-Carman equation was valid or not:

2.

(i). A cylindrical ion exchange bed composed of spherical particles 2 mm in diameter packed at a bed voidage of 0.45 is to be used to de-ionise a liquid of density and viscosity 1100 kg m^{-3} and 0.0075 Pa s respectively. The design flow rate is $5 \text{ m}^3 \text{ hour}^{-1}$ and the bed height and diameter are 2 and 0.2 m respectively, using the Kozeny-Carman equation the pressure drop is (Pa):

a: 99000 b: 1.32×10^7 c: 4400 d: 44000

(ii). The Modified Reynolds Number is:

a: 3.91 b: 4.78 c: 1290 d: 0.478

(iii). Comment on your use of the Kozeny-Carman equation:

(iv). The interstitial liquid velocity inside the bed is (m s^{-1}):

a: 0.020 b: 0.098 c: 0.08 d: 0.044

(v). Using the Carman correlation the shear stress on the ion exchange beads is (Pa):

a: 17.2 b: 3.5 c: 13.5 d: 27.0

(vi). Hence the dynamic pressure drop over the bed is (kPa):

a: 84 b: 130 c: 99 d: 150

(vii). Why is the answer to (vi) different to that in (i)?

(viii). If the liquid has a datum height equal to the position at the base of the ion exchange vessel and, therefore, needs raising to the top of the column before it enters the ion exchange bed the additional pressure drop to effect this, i.e. the static pressure drop over the bed, is (kPa):

a: 2.16 b: 21.6 c: 216 d: 0.22